EECS 861
Homework 6

1. $Z(t) = Xt + Y$

   Where X and Y are jointly Gaussian random variables with $E[X]=\mu_X=0$, $E[Y]=\mu_Y=0$, $\sigma_X=1$, $\sigma_y=1$, and $\rho_{XY}=0.5$.
   a. Find $P(Z(0)>1)$
   b. Find $E[Z(t)]$
   c. Find autocorrelation function, $R_{ZZ}(t_1,t_2)$
   d. Find autocovariance function, $C_{ZZ}(t_1,t_2)$
   e. Find the pdf of $Z(t)$.
   f. Find the joint pdf of $Z(0)$ and $Z(t_1)$.

2. Given $Z(t)$ in problem 1)
   a. Is $Z(t)$ strict sense stationary?
   b. Is $Z(t)$ wide sense stationary?

3. $X(t)$ and $Y(t)$ are SI WSS zero mean random process. Find the autocorrelation function of $Z(t)$:
   a. $Z(t) = 1 + 2X(t) + 3Y(t)$
   b. $Z(t) = X(t)Y(t)$
   c. Let $X(t)$ and $Y(t)$ be Gaussian SI WSS random processes with zero mean and unit variance, and as in part a) $Z(t) = 1 + 2X(t) + 3Y(t)$ find $P(Z(1)<0)$.

4. $Z(t) = X(t)-aX(t-d)$

   Where $a=.5$ and $d=1$ are constants, $E[X(t)]=\mu_X=0$ and the autocovariance of $X(t)$ is $C_{XX}(t_1,t_2)$.
   a. Find $E[Z(t)]$
   b. Find $R_{ZZ}(t_1,t_2)$
   c. Repeat b. assuming that $X(t)$ is a wide sense stationary random process.

5. For this problem use the data in these files
   http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2017/random_process_1.csv

   http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2017/random_process_2.csv

   http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2017/random_process_3.csv
   Each row is a member function of a discrete time random process X[n], Y[n], Z[n], one random process for each file.
   a) Consider row 1 column i and i+1 as a pair of random samples (e.g., X[i], X[i+1]); calculate and plot the correlation coefficient between these samples for all i. That is, column 1 & 2 is random sample one (e.g., X[1], X[2]), column 2 & 3 is random sample 2 (e.g., X[2], X[3]),, column 3 & 4 (e.g., X[3], X[4]), is random sample 3.
   b) Repeat part a) for column i and i+2.
   c) Repeat part a) for column i and i+3.
   d) Compare the results from above to your results from Homework 5 Problem 2.
   e) Comment on the stationary of these random processes.

6. A RV X_i is uniformly distributed between 100 and 200 and the X_i's are i.i.d. Let

$$y = \frac{1}{10}\sum_{i=1}^{10} X_i$$

a) Find E[Y]
b) Find Var[Y]
c) Find P(Y<140)